重回帰式の求め方

ここでは例として、目的変量が1つ、説明変量が6つの以下のものを考えます。
$y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_5 + b_6x_6 \ (= f(x_1,x_2,x_3,x_4,x_5,x_6))$
この回帰式について最小2乗法を当てはめていきます。

実際の値と回帰式との距離 $L = | y_i - f(x_{1i},x_{2i},x_{3i},x_{4i},x_{5i},x_{6i}) |$ の2乗和の最小値を求める。

実際には、偏微分
$\partial L / \partial b_0 = 0,\ \partial L / \partial b_1 = 0,\ \partial L / \partial b_2 = 0,\ \partial L / \partial b_3 = 0,$
$\partial L / \partial b_4 = 0,\ \partial L / \partial b_5 = 0,\ \partial L / \partial b_6 = 0$
を解けばよい。

$$\frac{\partial L}{\partial b_0} = \frac{\partial}{\partial b_0} \Sigma (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})^2 = 0$$
$$\frac{\partial L}{\partial b_1} = \frac{\partial}{\partial b_1} \Sigma (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})^2 = 0$$
$$\frac{\partial L}{\partial b_2} = \frac{\partial}{\partial b_2} \Sigma (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})^2 = 0$$
$$\frac{\partial L}{\partial b_3} = \frac{\partial}{\partial b_3} \Sigma (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})^2 = 0$$
$$\frac{\partial L}{\partial b_4} = \frac{\partial}{\partial b_4} \Sigma (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})^2 = 0$$
$$\frac{\partial L}{\partial b_5} = \frac{\partial}{\partial b_5} \Sigma (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})^2 = 0$$
$$\frac{\partial L}{\partial b_6} = \frac{\partial}{\partial b_6} \Sigma (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})^2 = 0$$

$$-2\Sigma (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i}) = 0$$
$$-2\Sigma (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i}) x_{1i} = 0$$
$$-2\Sigma (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i}) x_{2i} = 0$$
$$-2\Sigma (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i}) x_{3i} = 0$$
$$-2\Sigma (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i}) x_{4i} = 0$$
$$-2\Sigma (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i}) x_{5i} = 0$$
$$-2\Sigma (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i}) x_{6i} = 0$$

$\Sigma y_i = nb_0 + b_1 \Sigma x_{1i} + b_2 \Sigma x_{2i} + b_3 \Sigma x_{3i} + b_4 \Sigma x_{4i} + b_5 \Sigma x_{5i} + b_6 \Sigma x_{6i}$ ・・・①

$\Sigma x_{1i}y_i = b_0 \Sigma x_{1i} + b_1 \Sigma x_{1i}x_{1i} + b_2 \Sigma x_{1i}x_{2i} + b_3 \Sigma x_{1i}x_{3i} + b_4 \Sigma x_{1i}x_{4i} + b_5 \Sigma x_{1i}x_{5i} + b_6 \Sigma x_{1i}x_{6i}$ ・・・②

$\Sigma x_{2i}y_i = b_0 \Sigma x_{2i} + b_1 \Sigma x_{2i}x_{1i} + b_2 \Sigma x_{2i}x_{2i} + b_3 \Sigma x_{2i}x_{3i} + b_4 \Sigma x_{2i}x_{4i} + b_5 \Sigma x_{2i}x_{5i} + b_6 \Sigma x_{2i}x_{6i}$ ・・・③

$\Sigma x_{3i}y_i = b_0 \Sigma x_{3i} + b_1 \Sigma x_{3i}x_{1i} + b_2 \Sigma x_{3i}x_{2i} + b_3 \Sigma x_{3i}x_{3i} + b_4 \Sigma x_{3i}x_{4i} + b_5 \Sigma x_{3i}x_{5i} + b_6 \Sigma x_{3i}x_{6i}$ ・・・④

$\Sigma x_{4i}y_i = b_0 \Sigma x_{4i} + b_1 \Sigma x_{4i}x_{1i} + b_2 \Sigma x_{4i}x_{2i} + b_3 \Sigma x_{4i}x_{3i} + b_4 \Sigma x_{4i}x_{4i} + b_5 \Sigma x_{4i}x_{5i} + b_6 \Sigma x_{4i}x_{6i}$ ・・・⑤

$\Sigma x_{5i}y_i = b_0 \Sigma x_{5i} + b_1 \Sigma x_{5i}x_{1i} + b_2 \Sigma x_{5i}x_{2i} + b_3 \Sigma x_{5i}x_{3i} + b_4 \Sigma x_{5i}x_{4i} + b_5 \Sigma x_{5i}x_{5i} + b_6 \Sigma x_{5i}x_{6i}$ ・・・⑥

$\Sigma x_{6i}y_i = b_0 \Sigma x_{6i} + b_1 \Sigma x_{6i}x_{1i} + b_2 \Sigma x_{6i}x_{2i} + b_3 \Sigma x_{6i}x_{3i} + b_4 \Sigma x_{6i}x_{4i} + b_5 \Sigma x_{6i}x_{5i} + b_6 \Sigma x_{6i}x_{6i}$ ・・・⑦

これらの ①②③④⑤⑥⑦ の連立方程式を解けば、$b_0, b_1, b_2, b_3, b_4, b_5, b_6$ の値がわかるので、重回帰式が特定できる。